Exam 3 Review Problems

1. Consider the function $f(x,y) = \dfrac{x^3 y}{x^6 + y^2}$.

   a) Show that this function limits to zero along any straight line through the origin (you will need to consider horizontal lines, vertical lines and then lines of the form $y = mx$ with $m \neq 0$).

   b) Show that $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist. (HINT: Try paths of the form $y = x^n$.)

2. Prove that
$$\lim_{(x,y)\to(0,0)} \frac{x^4 + xy^3}{x^2 + y^2} = 0.$$
(HINT: Try polar coordinates.)

3. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ for the surface given by
$$x^2 y z^2 + z\cos(x+y) = 2x + 3y + z.$$

4. Find $\dfrac{dF}{dt}$ for the function $F(x,y,z) = xy + z^2$ where
$$\begin{aligned} x &= uv, \\ y &= u + v, \\ z &= u^2 - v^2, \\ u &= 2t + 1, \\ v &= \sin(2t). \end{aligned}$$

5. Find the equation of the tangent plane to the surface
$$x^2 y^2 z + z^2 = 6$$
at the point $(1,1,2)$.

6. Suppose that the elevation above sea-level in a particular region is given by the function
$$h(x,y) = x^2 y^2 + 2x^2.$$
A hiker is currently at position $(1,1)$.

   a) In what direction should the hiker go to climb uphill *as fast as possible*?

   b) Find the two directions the hiker can travel in order to *remain at the same elevation*.

7. Find and classify all critical points for the function
$$f(x,y) = 10xye^{-x^2-y^2}.$$

8. Find and classify the single critical point for the function

$$g(x, y, z) = 2xy + 3xz - 2yz.$$

(HINT: One of the eigenvalues for the Hessian matrix should be 3.)

9. Find the maximum and minimum values of the function $f(x, y) = xy$ in the region bounded by the ellipse

$$\frac{x^2}{8} + \frac{y^2}{2} = 1.$$

You must identify all relevant critical points inside the ellipse and any critical points on the surface.

10. Find the dimensions of the *open-topped box* with total surface area 3 that has maximum volume.